1972 AB2/BC1
Solution

$$x(t) = (t-2)^3(t-6)$$

$$v(t) = x'(t) = (t-2)^3 + 3(t-2)^2(t-6) = (t-2)^2((t-2)+3(t-6))$$
$$= (t-2)^2(4t-20) = 4(t-2)^2(t-5)$$

$$a(t) = x''(t) = 12(t-2)(t-4)$$

(a) The particle is moving to the right when $v(t) = 4(t-2)^2(t-5) > 0$. This happens for $t > 5$.

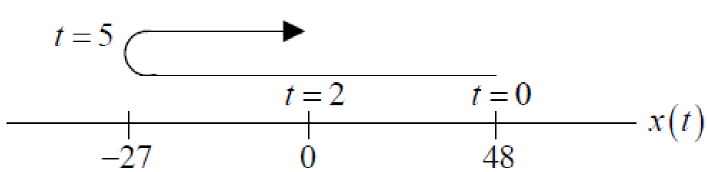
(b) The particle is at rest when $v(t) = 4(t-2)^2(t-5) = 0$. This happens at $t = 2$ and $t = 5$.

(c) The particle changes direction when the velocity changes sign. The velocity is negative just to the left and just to the right of $t = 2$. The velocity is negative for $t < 5$ and positive for $t > 5$. Therefore the particle only changes direction at $t = 5$.

(d) The minimum value of x is at $t = 5$ since the particle moves to the left for $t < 5$ and moves back to the right for $t > 5$.

$$x(5) = 3^3(-1) = -27$$

Therefore the farthest to the left of the origin that the particle moves is $x = -27$.



1975 AB2
Solution

(a) $v = \dfrac{dx}{dt} = t^2 - 6t + 8$

$v(0) = 8 > 0$ and so the particle is moving to the right at $t = 0$.

(b) The particle is moving to the left when $v(t) = t^2 - 6t + 8 = (t-4)(t-2) < 0$.
Therefore the particle moves to the left for $2 < t < 4$.

(c) At time $t = 3$, $x = \dfrac{1}{3}(3)^3 - 3(3)^2 + 8(3) = 6$.

(d) The particle changes direction at $t = 2$.

$x(0) = 0$

$x(2) = \dfrac{1}{3}(2)^3 - 3(2)^2 + 8(2) = \dfrac{20}{3}$

$x(3) = 6$

Distance $= (x(2) - x(0)) + (x(2) - x(3)) = \dfrac{20}{3} + \dfrac{2}{3} = \dfrac{22}{3}$

1975 AB4/BC1
Solution

(a) $y' = 1 + \cos x$

Therefore $x = \pi$ is the only critical point on the interval $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$. But $y' \ge 0$ on this interval, hence π is not an extreme point. The minimum and maximum must occur at the endpoints.

At $x = -\frac{\pi}{2}$, $y = -\frac{\pi}{2} + \sin\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2} - 1$

At $x = \frac{3\pi}{2}$, $y = \frac{3\pi}{2} + \sin\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - 1$

The absolute minimum is at $\left(-\frac{\pi}{2}, -\frac{\pi}{2} - 1\right)$.

The absolute maximum is at $\left(\frac{3\pi}{2}, \frac{3\pi}{2} - 1\right)$.

(b) $y'' = -\sin x$

$y'' = 0$ at $x = 0$ and $x = \pi$.

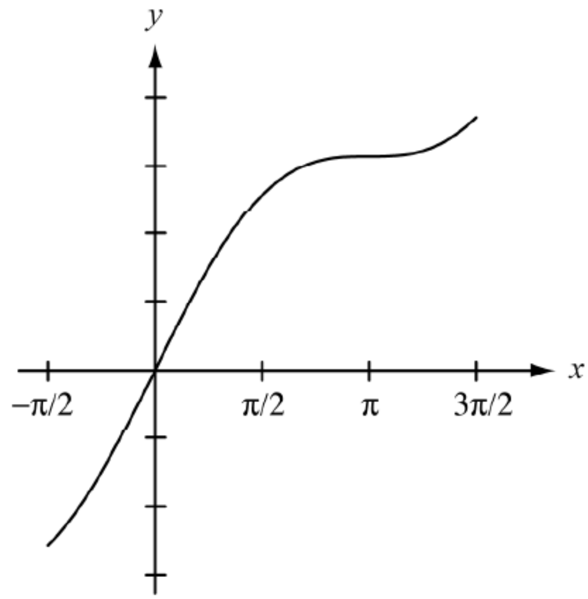
$y'' > 0$ for $-\frac{\pi}{2} < x < 0$

$y'' < 0$ for $0 < x < \pi$

$y'' > 0$ for $\pi < x < \frac{3\pi}{2}$

Therefore $(0,0)$ and (π, π) are inflection points.

(c)



1977 AB6
Solution

(a) <u>Implicit</u>

$$LW = 200$$

$$L\frac{dW}{dt} + W\frac{dL}{dt} = 0$$

$$(-0.5)L + 4W = 0$$
$$L = 8W$$
$$8W^2 = 200$$

$$W = \sqrt{\frac{200}{8}} = \sqrt{25} = 5$$

<u>Explicit</u>

$$W = \frac{200}{L}$$

$$\frac{dW}{dt} = -\frac{200}{L^2}\frac{dL}{dt}$$

$$-0.5 = -\frac{200}{L^2}\cdot 4$$
$$L^2 = 1600 \text{ so } L = 40$$

$$W = \frac{200}{40} = 5$$

(b) <u>Implicit with W</u>

$$D^2 = L^2 + W^2$$

When $W = 10$, $L = 20$ and $D^2 = 500$

$$2D\frac{dD}{dt} = 2L\frac{dL}{dt} + 2W\frac{dW}{dt}$$

From (a),
$$\frac{dW}{dt} = -\frac{W}{L}\cdot\frac{dL}{dt} = -\frac{10}{20}\cdot 4 = -2$$

$$\frac{dD}{dt} = \frac{1}{10\sqrt{5}}\big(20\cdot 4 + 10(-2)\big)$$

$$\frac{dD}{dt} = \frac{6}{\sqrt{5}}$$

<u>Explicit</u>

$$D = \sqrt{L^2 + W^2}$$

$$D = \sqrt{L^2 + \left(\frac{200}{L}\right)^2}$$

$$\frac{dD}{dt} = \frac{2L - \frac{2(200)^2}{L^3}}{2\sqrt{L^2 + \left(\frac{200}{L}\right)^2}}\cdot\frac{dL}{dt}$$

$$\frac{dD}{dt} = \frac{2\cdot 20 - 10}{2\sqrt{20^2 + 10^2}}\cdot 4$$

$$\frac{dD}{dt} = \frac{6}{\sqrt{5}}$$